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1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

2022

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–1

The figures in the margin indicate full marks for the questions

Paper : GE–1 (A)

(Differential Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* কেতিয়া এটা ফলন f বন্ধ অন্তৰ $[a, b]$ত অনৱচ্ছিন্ন হোৱা বুলি কোৱা হয়? 1

When is a function f said to be continuous in a closed interval $[a, b]$?

(b) তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 3

Evaluate any *one* of the following :

(i) $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(ii) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(c) f ফলনৰ সংজ্ঞা এনেদৰে দিয়া আছে

$$f(x) = (1+3x)^{1/x}, \quad x \neq 0$$
$$= e^3 \quad , \quad x = 0$$

দেখুওৱা যে $x=0$ বিন্দুত ফলন অনৱচ্ছিন্ন। 3

Show that the function f defined by

$$f(x) = (1+3x)^{1/x}, \quad x \neq 0$$
$$= e^3 \quad , \quad x = 0$$

is continuous at $x = 0$.

(d) $y=(ax+b)^m$ৰ n-তম অৱকলজ নিৰ্ণয় কৰা য'ত $n \le m$ আৰু $m, n \in N$. 1

Find the n-th derivative of $y=(ax+b)^m$, where $n \le m$ and $m, n \in N$.

(e) যদি (If)

$$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

দেখুওৱা যে (show that)

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$$ 4

2. লিবনিটজৰ উপপাদ্যটো উল্লেখ কৰা আৰু প্ৰমাণ কৰা। 5

State and prove Leibnitz's theorem.

অথবা / Or

যদি (If)

$$u = \tan^{-1} \frac{x^3+y^3}{x-y}$$

তেন্তে প্ৰমাণ কৰা যে (then prove that)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

3. (a) যদি $u = f(xyz)$ হয়, তেন্তে $\frac{\partial f}{\partial y}$ নিৰ্ণয় কৰা। 1

If $u = f(xyz)$, then find $\frac{\partial f}{\partial y}$.

(b) যদি (If)

$$u = \sin^{-1}\left\{\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right\}$$

তেন্তে প্ৰমাণ কৰা যে (then prove that)

$$\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$$ 4

(c) যদি $y = \sin^2 x$, তেন্তে y_n নিৰ্ণয় কৰা। 1

If $y = \sin^2 x$, then find y_n.

4. (a) যদি $f = \tan^{-1}\frac{y}{x}$ হয়, তেন্তে $\frac{\partial f}{\partial x}$ নিৰ্ণয় কৰা। 1

If $f = \tan^{-1}\frac{y}{x}$, then find $\frac{\partial f}{\partial x}$.

(b) দেখুওৱা যে এটা ফলন $f(x) = |x| + |x-1|$, এটা বিন্দু $x = 1$ ত অনৱচ্ছিন্ন কিন্তু অৱকলনীয় নহয়। 3

Show that the function f defined as follows, is continuous but not derivable at $x = 1$, $f(x) = |x| + |x-1|$.

(c) যদি (If)

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

তেন্তে দেখুওৱা যে (then show that)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

3

5. (a) $y = x^2(a-x)$ বক্ৰৰ উপস্পৰ্শকৰ দৈৰ্ঘ্য নিৰ্ণয় কৰা। 1

Find the length of the subtangent to the curve $y = x^2(a-x)$.

(b) দেখুওৱা যে, যি কোনো বক্ৰৰ ক্ষেত্ৰত

$$\frac{\text{উপ-অভিলম্ব}}{\text{উপ-স্পৰ্শক}} = \left(\frac{\text{অভিলম্বৰ দীঘ}}{\text{স্পৰ্শকৰ দীঘ}}\right)^2$$

2

Show that in any curve

$$\frac{\text{subnormal}}{\text{subtangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$

6. (a) যি কোনো বক্ৰৰ ক্ষেত্ৰত উপস্পৰ্শকৰ সংজ্ঞা লিখা। 1

Define subtangent to any curve.

(b) $x = a(\theta + \sin\theta)$ আৰু $y = a(1 - \cos\theta)$ বক্ৰৰ θ ত উপস্পৰ্শকৰ দৈৰ্ঘ্য নিৰ্ণয় কৰা। 3

Find the lengths of subtangent to $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ at θ.

7. তলত দিয়া বক্ৰৰ অনন্তস্পৰ্শী নিৰ্ণয় কৰা : 4

Find the asymptotes of the following curve :

$$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$$

অথবা / *Or*

$a^4y^2 = x^4(2x^2 - 3a^2)$ বক্ৰৰ অৱস্থান আৰু দ্বি-বিন্দুৰ প্ৰকৃতি নিৰ্ণয় কৰা।

Find the position and nature of the double points of the curve $a^4y^2 = x^4(2x^2 - 3a^2)$.

8. তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 4

Evaluate any *one* of the following :

(a) $y = x(x^2 - 1)$ বক্ৰৰ অনুৰেখন নিৰ্ণয় কৰা।

Trace the curve $y = x(x^2 - 1)$.

(b) দেখুওৱা যে $r = a(1 - \cos\theta)$ কাৰ্ডিয়ইডৰ যি কোনো বিন্দু (r, θ) ত বক্ৰতা ব্যাসাৰ্ধ $\frac{2}{3}\sqrt{2ar}$.

Show that the radius of curvature at any point (r, θ) of the cardioid $r = a(1 - \cos\theta)$ is given by $\frac{2}{3}\sqrt{2ar}$.

9. $f(x, y) = 0$ বক্ৰৰ যি কোনো বিন্দু (x, y)ত বহু বিন্দু হোৱাৰ প্ৰয়োজনীয় আৰু পৰ্যাপ্ত চৰ্ত উল্লেখ কৰি প্ৰমাণ কৰা। 5

State and prove the necessary and sufficient condition for any point (x, y) on the curve $f(x, y) = 0$ to be a multiple point.

অথবা / Or

এটা বক্ৰৰ কাৰ্টেচিয়ান সমীকৰণ $y = f(x)$ হ'লে বক্ৰৰ এটা বিন্দুত বক্ৰতা ব্যাসাৰ্ধ নিৰ্ণয় কৰা।

Find the radius of curvature at a point of the Cartesian equation of the curve $y = f(x)$.

10. *(a)* ৰোলৰ উপপাদ্যটো লিখা। 1

State the Rolle's theorem.

(b) $[-1, 1]$ অন্তৰালত $f(x) = \frac{1}{2-x^2}$ ফলনৰ বাবে ৰোলৰ উপপাদ্য প্ৰতিপন্ন কৰা। 2

Verify Rolle's theorem for the function

$$f(x) = \frac{1}{2-x^2}$$

in the interval $[-1, 1]$.

(c) মধ্যমান উপপাদ্য $f(b) - f(a) = (b-a)f'(\xi)$ প্ৰতিপন্ন কৰা য'ত $f(x) = x(x-1)(x-3)$, $a = 0$, $b = \frac{1}{2}$ আৰু ξ ৰ মান নিৰ্ণয় কৰা। 4

Verify the applicability of the mean value theorem $f(b) - f(a) = (b-a)f'(\xi)$, $a < \xi < b$ if $f(x) = x(x-1)(x-3)$, where $a = 0$, $b = \frac{1}{2}$. Also find the value of ξ.

11. লাগ্ৰাঞ্জৰ মধ্যমান উপপাদ্য উল্লেখ কৰি প্ৰমাণ কৰা। 1+4=5

State and prove Lagrange's mean value theorem.

অথবা / Or

মেক্লৰিনৰ উপপাদ্য ব্যৱহাৰ কৰি $\sin x$ক x-ৰ সূচকত অসীম শ্ৰেণীত বিস্তৃতি কৰা। 5

Using Maclaurin's theorem, expand $\sin x$ in an infinite series in powers of x.

12. *(a)* যদি (If)

$$f(x) = f(0) + xf'(0) + \frac{x^2}{\underline{|2}} f''(\theta x)$$

তেন্তে θৰ মান উলিওৱা যেতিয়া $x \to 1$ আৰু য'ত $f(x) = (1-x)^{5/2}$.

then find θ when $x \to 1$ and where $f(x) = (1-x)^{5/2}$. 3

(b) $f(x, y) = x^3 + y^3 - 3x - 12x + 20$ ফলনৰ সৰ্বোচ্চ আৰু সৰ্বনিম্ন মান নিৰ্ণয় কৰা। 4

Find the maximum and minimum values of the function

$$f(x, y) = x^3 + y^3 - 3x - 12x + 20$$

13. *(a)* $\log x$ক $x-1$ৰ সূচকত বিস্তৃতি কৰা য'ত $0 < x \le 2$. 1

Expand $\log x$ in powers of $x-1$ where $0 < x \le 2$.

(b) তলৰ যি কোনো এটাৰ মান নিৰ্ণয় কৰা : 4

Evaluate any *one* of the following :

(i) $\lim_{x \to 1} \left\{ \frac{x}{x-1} - \frac{1}{\log x} \right\}$

(ii) $\lim_{x \to 0} (\cos x)^{\cot^2 x}$

14. *(a)* লাগ্ৰাঞ্জৰ ৰূপৰ অৱশেষ থকা মেক্‌লৰিনৰ উপপাদ্য লিখা। 1

Write the Maclaurin's theorem with Lagrange's form of remainder.

(b) মেক্‌লৰিনৰ অসীম শ্ৰেণী ব্যৱহাৰ কৰি $\log(1+x)$ৰ বিস্তৃতি কৰা য'ত $-1 < x < 1$. 5

Expand $\log(1+x)$ using Maclaurin's infinite series where $-1 < x < 1$.

অথবা / *Or*

লাগ্ৰাঞ্জৰ ৰূপৰ অৱশেষ থকা টেইলৰৰ উপপাদ্য লিখি প্ৰমাণ কৰা।

State and prove Taylor's theorem with Lagrange's form of remainder.

Paper : GE–1 (B)

(Object-Oriented Programming in C++)

Full Marks : 60
Pass Marks : 24

Time : 3 hours

1. Answer the following questions : 1×5=5

(a) Define abstraction.

(b) State one difference between C and C++.

(c) Write one characteristic of object-oriented programming language.

(d) What is the use of <iostream.h>?

(e) How are objects created from a class?

2. Answer any *five* of the following questions : 2×5=10

(a) When do you declare a method or class abstract?

(b) Briefly explain the structure of C++ program.

(c) How does inheritance help us to create new classes?

(d) Why can we not override static method?

(e) State the difference between while loop and do while loop.

(f) Define default constructor and copy constructor.

3. Answer any *five* of the following questions : 3×5=15

(a) Explain the following operators and their uses :

cin, cout and delete.

(b) Explain the three access modifiers.

(c) What is dynamic binding? Define message passing.

(d) State the difference between break and continue with example.

(e) Define file pointer. What is function prototyping? Explain with example.

(f) Explain the increment and decrement operators in brief.

4. Answer any *four* of the following questions : 5×4=20

(a) Write a C++ program to store information of a book in a structure.

(b) Write a C++ program to overload a unary operator.

(c) Write a C++ program to display Fibonacci series up to 50.

(d) Write a C++ program to implement friend function.

(e) Write a C++ program to count the number of objects created.

5. *(a)* Explain the different types of inheritance with examples and diagrams. 10

Or

(b) Explain inline and virtual functions with suitable example.

Paper : GE–1 (C)

(Finite Element Methods)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. *(a)* Write True or False : 1

The finite-element method is a piecewise application of a variational method.

(b) Write down the differences between finite difference methods and finite element methods. 3

(c) Consider the boundary value problem

$$u'' + (1 + x^2)u + 1 = 0$$

Determine the coefficients of the approximate solution

$$W(x) = a_1(1 - x^2) + a_2 x^2 (1 - x^2)$$

by using the least square method. 5

Or

Using Galerkin's method, solve the boundary value problem

$$\nabla^2 u = -1, \quad |x| \le 1, \quad |y| \le 1$$
$$u = 0 , \quad |x| = 1, \quad |y| = 1$$

with $h = \frac{1}{2}$.

(d) Find the variational functional for the boundary value problem

$$u'' = u + 4xe^{x}$$
$$u'(0) - u(0) = 1, \quad u'(1) + u(1) = -e$$

5

(e) State and prove the Lax-Milgram theorem. 6

2. *(a)* The application of the finite element method to the boundary value problem

$$-u'' = x$$
$$u(0) = u(1) = 0$$

leads to the system of equations $Au = b$. Determine the matrix A and the column vector b for four elements of equal lengths. 6

(b) Apply Galerkin method to the boundary value problem

$$\nabla^2 u + \lambda u = 0, \quad |x| \le 1, \quad |y| \le 1$$
$$u = 0, \quad |x| = 1, \quad |y| = 1$$

to get the characteristic equation in the form $|A - \lambda B| = 0$. 6

3. *(a)* Define assembly of the element equations. 1

(b) Define two principles that were used in one-dimensional problem to assembly of finite element equations. 2

(c) Discuss briefly with an example about the element assemblage in finite element method. 3

(d) Write down the importance of sparse matrix in the process of element assemblage with an example. 4

(e) If the finite solutions at any point (x, y) in an element Ω^e is given by

$$U(x, y) = \sum_{J=1}^{n} U_J^e \psi_J^e (x, y)$$

Find its derivatives. 2

4. *(a)* State the properties for a quadratic triangular element. 3

(b) Give an example of triangular element with a common node. 1

(c) Illustrate the process of discretization in two-dimensional domain with a suitable example. 5

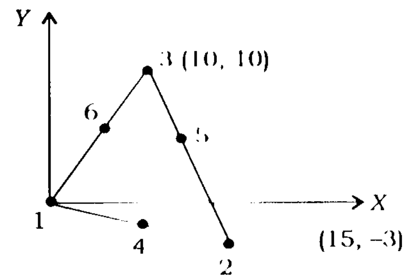
(d) Write the importance of isoperimetric element in the process of element assemblage with an example. 3

5. *(a)* Write True or False : 1

Finite element modelling involves assumptions concerning the representation of the system and its behaviour.

(b) Write about interpolating function in finite element method. Find an expression for interpolating function in one-dimensional domain. 3

(c) Calculate the interpolation function for the quadratic triangular element shown in the figure : 4



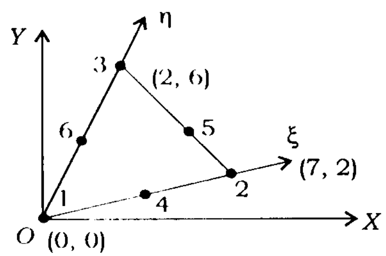
(d) Evaluate the integral of the form

$$I = \int_{(e)} F(x)\, dx$$

for the triangular element where $F(x)$ is given function, (e) is the element and x represents multidimensional coordinates. 4

Or

Consider the quadratic triangular element shown in the figure :



Evaluate the integral of the product

$$\left(\frac{\partial\psi_1}{\partial x}\right)\left(\frac{\partial\psi_4}{\partial x}\right)$$

at the point $(x, y) = (2, 4)$.

6. *(a)* What are the different types of partial differential equations? Write their field in applications. 4

(b) Find the solution of the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1+e^u}{2} = 0\,, \quad |x| \le 1, \quad |y| \le 1$$

$$u = 0\,, \quad |x| = 1, \quad |y| = 1$$

by finite element method (use the three node triangular element). 4

(c) Use finite element method to solve the boundary value problem

$$\nabla^2 u = -1, \quad |x| \le 1, \quad |y| \le 1$$

$$\frac{\partial u}{\partial x} + u = 0\,, \quad |x| = 1, \quad |y| = 1$$

with $h = \frac{1}{2}$. 4

★★★