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1 SEM TDC MTMH (CBCS) C 1

2022

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the value of $\frac{d}{dx}\tanh x$. 1

(b) Write the curve on which the point $(\cosh x, \sinh x)$ lies. 1

(c) Write the interval on which 'secant' is one-to-one. 1

(d) Find y_n, if $y = \sin 5x \cos 2x$. 2

(e) Find y_n, if $y = x^3 \sin x$. 3

(f) Sketch the general shape of the graph of $y = f(x)$, where $\frac{dy}{dx} = 2 + x - x^2$. 3

(g) Find y_n, if $y = e^{ax+b} \sin x$. 4

Or

Evaluate $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$.

(h) Find the asymptotes of the curve

$$y^2 - x^2 - 2x - 2y - 3 = 0$$ 5

Or

For the curve $y = x + \sin 2x$, $-\frac{2\pi}{3} \le x \le \frac{2\pi}{3}$, find the local maximum, local minimum and the interval on which the curve is concave up and concave down.

2. *(a)* Write the washer's area with outer radius $R(x)$ and inner radius $r(x)$. 1

(b) Obtain the reduction formula for $\int x^n e^{-ax} dx$. 4

(c) Obtain the reduction formula for $\int \cos^n x \, dx$. 5

Or

Find $\int \tan^4 x \, dx$.

(d) Find the value of $\int_0^1 \frac{\sin^3 x}{\cos^6 x} dx$. 5

Or

Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the line $y = 0$, $x = 2$, about x-axis.

3. *(a)* Write the parametrization of the graph of the function $f(x) = x^2$. 1

(b) If a curve is symmetric about x-axis and the point (r, θ) lies on the graph, then write which of the following also lies on the graph : 1

(i) $(r, \pi - \theta)$

(ii) $(-r, \pi - \theta)$

(iii) $(-r, -\theta)$

(iv) $(-r, \theta)$

(c) Define a parametric curve. 2

(d) Write the polar equation of $xy = 1$. 1

(e) Write the equivalent Cartesian equation of $r^2 \sin 2\theta = 2$. 2

(f) Find the perimeter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is defined parametrically by $x = a \sin t$, $y = b \cos t$, $a > b$ and $0 \le t \le 2\pi$. 4

Or

Find the centroid of the first-quadrant arc of the asteroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \le t \le 2\pi$.

(g) Find the length of the curve $x = \cos t$, $y = t + \sin t$, $0 \le t \le \pi$. 4

Or

Find the centre, foci, vertices of the conic section $x^2 + 2x + 4y - 3 = 0$.

4. (a) Define a vector function. 1

(b) Write the value of $(\vec{u} \times \vec{v}) \cdot \vec{v}$. 1

(c) Define triple scalar product of vectors. 2

(d) Show that vector and its first derivative are orthogonal. 3

Or

Evaluate $\int_0^1 (te^{t^2}\hat{i} + e^{-t}\hat{j} + \hat{k})\,dt$.

(e) Find the unit tangent vector of the curve $\vec{r}(t) = \sin 2t\,\hat{i} + \cos 2t\,\hat{j} + \hat{k}$, $0 \le t \le \pi$. 3

Or

Find the acceleration of the particle described by $\vec{r} = (t-1)\hat{i} + (t^2 - 1)\hat{j} + 2t\hat{k}$ at $t = 1$.

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