VI\_15: **Reciprocal lattice to fcc lattice:**

 If we draw a fcc lattice and consider three face centered lattice point P, Q and R, we will get the translational lattice vectors $\vec{a}$ , $\vec{b}$ and $\vec{c}$ as

$\vec{a}$ = $\frac{a}{2}$ ($\hat{i}$ + $\hat{j}$ ), $\vec{b}$ = $\frac{a}{2}$ ($\hat{j}$ + $\hat{k}$ ) and $\vec{c}$ = $\frac{a}{2}$ ($\hat{k}$ + $\hat{i}$ )

(\* *See the box 2 in the previous note).*

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 Also, the reciprocal translational vectors are given by:

 $\vec{A}$ or a\* = 2л $\frac{(\vec{b} x \vec{c})}{\vec{a} . (\vec{b} x \vec{c}) }$ , $\vec{B}$ or b\* = 2л $\frac{(\vec{c} x \vec{a})}{ \vec{b} . (\vec{c} x \vec{a})) }$ and $\vec{C}$ or c\* = 2л $\frac{(\vec{a}x \vec{b})}{ \vec{c} . ((\vec{a}x \vec{b}) }$



 Now,

 $(\vec{b} x \vec{c})$ = $\frac{a}{2}$ ($ \hat{j}$ + $\hat{k}$ ) x $\frac{a}{2}$ ($\hat{k}$ + $\hat{i}$ ) = ( a2 /4 ) [ (($ \hat{j}$ + $\hat{k}$ ) x ($\hat{k}$ + $\hat{i}$ ) ]

 If we make a determinant with the multiplying factors of the unit vectors, we get

 

 = $\hat{i}$ ( 1 – 0 ) - $\hat{j}$ (0 – 1 ) + $\hat{k}$ ( 0 – 1 )

 = $\hat{i}$ + $\hat{j}$ - $\hat{k}$



 So, $(\vec{b} x \vec{c})$ = ( a2 /4) ($\hat{i}$ + $\hat{j}$ - $\hat{k}$ )

 And,



Again,

 $(\vec{c} x \vec{a})$ = $\frac{a}{2}$ ($ \hat{k}$ + $\hat{i}$ ) x $\frac{a}{2}$ ($\hat{i}$ + $\hat{j}$ ) = (a2/4) [($ \hat{k}$ + $\hat{i}$ ) x ($\hat{i}$ + $\hat{j}$ ) ]

Making again a determinant, we have

 

 = $\hat{i}$ ( 0 – 1 ) - $\hat{j}$ ( 0 – 1 ) + $\hat{k}$ ( 1 – 0 )

 = - $\hat{i}$ + $\hat{j}$ + $\hat{k}$

So,

 $\vec{c} x \vec{a}$ = (a2/4) (- $\hat{i}$ + $\hat{j}$ + $\hat{k}$ ) and $\vec{b}$ . ($\vec{c} x \vec{a}$ ) = a3/4.

Hence, $\vec{B}$ or b\* = $\frac{2л}{a}$ (- $\hat{i}$ + $\hat{j}$ + $\hat{k}$ ) …. ……….. ………. ……… (2)

Similarly, we can determine

 $\vec{C}$ or c\* = $\frac{2л}{a}$ ( $\hat{i}$ - $\hat{j}$ + $\hat{k}$ ) … … … … … … … (3)

 The vectors a\*, b\* and c\* are the reciprocal lattice vectors of fcc lattice. These are also the primitive translation vectors of a bcc lattice (real lattice). Thus, the reciprocal lattice to an fcc lattice is a bcc lattice.

 