10. Calculate the co-efficient of correlation r_{xy} from the following data :

$\sum X = 71,\ \sum Y = 70,\ \sum X^2 = 555,\ \sum Y^2 = 526,\ \sum XY = 527,$ n = 10

Solution : We know that, $r_{xy} = \frac{Cov\ (X,Y)}{\sigma_x \sigma_y}$

$$\therefore \qquad r_{xy} = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{n\sum X^2 - (\sum X)^2}\sqrt{n\sum Y^2 - (\sum Y)^2}}$$

$$= \frac{10 \text{ x } 527 - 71 \text{ x } 70}{\sqrt{10 \text{ x } 555 - (71)^2}\ \sqrt{10 \text{ x } 526 - (70)^2}}$$

$$= \frac{5270 - 4970}{\sqrt{509}\ \sqrt{360}}$$

$$= \frac{5270 - 4970}{\sqrt{509}\ \sqrt{360}}$$

= 0.70

11. Find the co-efficient of correlation between X and Y from the following data :

X	:	2	4	5	6	8	11
Y	:	18	12	10	8	7	2

Solution : The table is prepared with given data as :

X	Y	U = X – 6	V = Y – 8	U^2	V^2	UV
2	18	–4	10	16	100	–40
4	12	–2	4	4	16	–8
5	10	–1	2	1	4	–2
6	8	0	0	0	0	0
8	7	2	–1	4	1	–2
11	2	5	–6	25	36	–30
		$\sum U = 0$	$\sum V = 9$	$\sum U^2 = 50$	$\sum V^2 = 157$	$\sum UV = -82$

Here, we have, $r_{xy} = r_{uv} = \frac{n\sum UV - (\sum U)(\sum V)}{\sqrt{n\sum U^2 - (\sum U)^2}\sqrt{n\sum V^2 - (\sum V)^2}}$

$$= \frac{6 \text{ x } (-82) - 0 \text{ x } 9}{\sqrt{6 \text{ x } 50 - 0}\ \sqrt{6 \text{ x } 157 - 81}}$$

$$= \frac{-492}{508.23} \quad = -0.97$$

12. Find the co-efficient of correlation between X and Y from the following data and interpret the result.

X	:	16	20	24	28	32
Y	:	30	40	25	35	45

Solution : Here, we have, $\bar{X} = \frac{120}{5} = 24, \quad \bar{Y} = \frac{175}{5} = 35$

Since, $\bar{X}$ and $\bar{Y}$ are whole numbers, we can proceed as follows :

X	Y	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$	$(X - \bar{X})\ (Y - \bar{Y})$
16	30	–8	–5	64	25	40
20	40	–4	5	16	25	–20
24	25	0	–10	0	100	0
28	35	4	0	16	0	0
32	45	8	10	64	100	80
$\sum X = 120$	$\sum Y = 175$			$\sum(X - \bar{X})^2$=160	$\sum(Y - \bar{Y})^2$=250	$\sum(X - \bar{X})(Y - \bar{Y}) = 100$

$$\therefore \quad r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2 \sum(Y-\bar{Y})^2}} = \frac{100}{\sqrt{160 \times 250}} = \frac{100}{200} = 0.5$$

Interpretation : Since, $r = 0.5$, we find positive correlation between the variables X and Y.

13. Calculate the co-efficient of correlation from the following data :

$n = 10, \quad \sum x = 140, \quad \sum y = 150, \quad \sum(x - 10)^2 = 180,$

$\sum(y - 15)^2 = 215, \quad \sum(x - 10)(y - 15) = 60$

Solution : Let us take, $u = x - 10, \quad v = y - 15$

Then, we have, $\sum u = \sum(x - 10) = \sum x - n \times 10 = 140 - 100 = 40$

$$\sum v = \sum(x-15) = \sum y - n \times 15 = 150 - 150 = 0$$

$$\sum u^2 = \sum(x-10)^2 = 180$$

$$\sum v^2 = \sum(x-15)^2 = 215$$

$$\sum uv = \sum(x-10)(y-15) = 60$$

$$\therefore \quad r_{xy} = r_{uv} = \frac{n\sum uv - (\sum u)(\sum v)}{\sqrt{n\sum u^2 - (\sum u)^2}\sqrt{n\sum v^2 - (\sum v)^2}}$$

$$= \frac{10 \text{ x } 60 - 40 \text{ x } 0}{\sqrt{10 \text{ x } 180 - (40)^2}\ \sqrt{10 \text{ x } 215 - 0}}$$

$$= \frac{600}{\sqrt{200 \times 2150}} = \frac{6}{6.557} = 0.91$$

14. Show that the correlation co-efficient between x and $a-x$ is -1. **(Important)**

Solution : We know that $r_{xy} = \frac{Cov\ (x,y)}{\sigma_x \sigma_y}$

$$\therefore \quad r_{x(a-x)} = \frac{Cov\ (x,a-x)}{\sigma_x \sigma_{a-x}}$$

$$= \frac{\frac{1}{n}\sum(x-\bar{x})(a-x-a+\bar{x})}{\sqrt{\frac{1}{n}\sum(x-\bar{x})^2}\ \sqrt{\frac{1}{n}\sum(a-x-a+\bar{x})^2}}$$

$$= \frac{-\sum(x-\bar{x})^2}{\sum(x-\bar{x})^2} = -1$$

15. Given that $r_{xy} = 0.6$, $cov(x,y) = 7.2$, $var(y) = 16$, find the standard deviation x. **(Important)**

Solution : We know that

$$r_{xy} = \frac{Cov\ (x,y)}{\sigma_x \sigma_y}$$

⇨ $0.6 = \frac{7.2}{\sigma_x \sqrt{16}}$ since SD = $\sqrt{var}$

⇨ $\sigma_x = \frac{7.2}{0.6 \times 4}$

⇨ $\sigma_x = 3$