16. Find the SD of the following numbers :

27, 60, 40, 30, 32

<u>Solution</u> :

x	x^2
27	729
60	3600
40	1600
30	900
32	1024
$\sum x = 189$	$\sum x^2 = 7853$

$$\bar{x} = \frac{\sum x}{n} = \frac{189}{5} = 37.8$$

$$\sigma = \sqrt{\frac{1}{n}\sum x^2 - \bar{x}^2} = \sqrt{\frac{7853}{5} - (37.8)^2}$$
$$= \sqrt{1570.6 - 1428.84}$$
$$= 11.9$$

<u>Using short-cut Method</u> :

x	$d = x - 40$	d^2
27	–13	169
60	20	400
40	0	0
30	–10	100
32	–8	64
	$\sum d = -11$	$\sum d^2 = 733$

$$\sigma = \sqrt{\frac{1}{n}\sum d^2 - \left(\frac{1}{n}\sum d\right)^2}$$
$$= \sqrt{\frac{733}{5} - \left(\frac{-11}{5}\right)^2}$$
$$= \sqrt{146.6 - 4.84}$$
$$= 11.9$$

17. Find SD from the following :

Items :	5	15	25	35	40
Frequency :	5	8	15	16	6

<u>Solution</u> :

x	f	$d = x - A$	fd	fd^2
5	5	–20	–100	2000
15	8	–10	–80	800
25	15	0	0	0
35	16	10	160	1600
40	6	15	90	1350
	$\sum f$= N = 50		$\sum fd = 70$	$\sum fd^2 = 5750$

$$\sigma = \sqrt{\frac{1}{N}\sum fd^2 - \left(\frac{1}{N}\sum fd\right)^2} = \sqrt{\frac{5750}{50} - \left(\frac{70}{50}\right)^2}$$

$$= \sqrt{115 - 1.96} = 10.63$$

18. Calculate SD from the following data :

Class Interval :	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency :	3	61	223	137	53	19	4

<u>Solution</u> :

Class Interval	f	x	d'	fd'	fd'^2
10-19	3	14.5	–3	–9	27
20-29	61	24.5	–2	–122	244
30-39	223	34.5	–1	–223	223
40-49	137	44.5	0	0	0
50-59	53	54.5	1	53	53
60-69	19	64.5	2	38	76
70-79	4	74.5	3	12	36
	N = 500			$\sum fd' = -251$	$\sum fd'^2 = 659$

$$\sigma = \sqrt{\frac{1}{N}\sum fd'^2 - \left(\frac{1}{N}\sum fd'\right)^2} \times h = \sqrt{\frac{659}{500} - \left(\frac{-251}{500}\right)^2} \times 10$$

$$= \sqrt{1.318 - 0.252} \times 10 = 1.032 \times 10 = 10.32$$

19. Find the mean deviation from the mean of the following data :

Size of items x_i	4	6	8	10	12	14	16
Frequency f_i	2	4	5	3	2	1	4

Solution : Here mean is $\bar{x} = \frac{\sum x_i}{7} = 10$

x_i	f_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
4	2	–6	6	12
6	4	–4	4	16
8	5	–2	2	10
10	3	0	0	0
12	2	2	2	4
14	1	4	4	4
16	4	6	6	24
	21			70

Mean deviation from Mean $= \frac{\sum_{i=1}^{n} f_i\lvert x-\bar{x}\rvert}{N} = \frac{70}{21} = 3.33$

20. Calculate the mean deviation from mean of the following distribution :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	5	8	15	16	6

Solution : Here mean is $\bar{x} = \frac{\sum x_i}{5} = 25$

Marks	Class Marks x_i	f_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0 – 10	5	5	–20	20	100
10 – 20	15	8	–10	10	80
20 – 30	25	15	0	0	0
30 – 40	35	16	10	10	60
40 – 50	45	6	20	20	120
		50			360

Mean deviation from Mean $= \frac{\sum_{i=1}^{n} f_i\lvert x-\bar{x}\rvert}{N} = \frac{360}{50} = 7.2$

Try yourself (Home Work) :

21. The ages of 10 girls are given below :

3 5 7 8 9 10 12 14 17 18

What is the range ?

22. The weight of 10 students (in Kg) of class XII are given below :

45 49 55 43 52 40 62 47 61 58

What is the range ?

23. Find the mean deviation from mean of the data

45 55 63 76 67 84 75 48 62 65

Given mean = 64.

24. Calculate the mean deviation from mean of the following distribution.

Salary (in rupees)	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 – 80	80 - 90	90 - 100
No. of employees	4	6	8	12	7	6	4	3

25. Calculate the mean deviation from mean for the following data of marks obtained by 40 students in a test :

Marks Obtained	20	30	40	50	60	70	80	90	100
No. of students	2	4	8	10	8	4	2	1	1

26. The data below presents the earnings of 50 workers of a factory :

Earnings (in rupees)	1200	1300	1400	1500	1600	1800	2000
No. of workers	4	6	15	12	7	4	2

Find mean deviation from mean.

27. The distribution of weight of 100 students is given below :

Weight (in Kg)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
No. of students	5	13	35	25	17	5

Calculate the mean deviation from mean.

28. The marks of 50 students in a particular test are :

Marks	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 – 80	80 - 90	90 - 100
No. of students	4	6	9	12	8	6	4	1

Find the Mean Deviation from above data.

29. Prove that SD is the minimum value of root mean square deviation. (1st property of SD)

Solution : For a frequency distribution SD is,

$$\sigma = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2}$$

Root mean square deviation is

$$s = \sqrt{\frac{1}{N}\sum f_i(x_i - A)^2}$$ where A is any arbitrary number.

$$\therefore \quad s^2 = \frac{1}{N}\sum f_i(x_i - A)^2$$

$$= \frac{1}{N}\sum f_i(x_i - \bar{x} + \bar{x} - A)^2$$

$$= \frac{1}{N}\sum f_i[(x_i - \bar{x})^2 + (\bar{x} - A)^2 + 2(x_i - \bar{x})(\bar{x} - A)]$$

$$= \frac{1}{N}\sum f_i(x_i - \bar{x})^2 + (\bar{x} - A)^2\frac{1}{N}\sum f_i + 2(\bar{x} - A)\frac{1}{N}\sum f_i\,(x_i - \bar{x})$$

But, $\sum f_i\,(x_i - \bar{x}) = 0$, by the properties of AM

$$\therefore \quad s^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2 + (\bar{x} - A)^2$$

$$\Rightarrow \quad s^2 = \sigma^2 + d^2; \;\; d = \bar{x} - A$$

s^2 will be least when $d = 0$

$$\Rightarrow \quad \bar{x} - A = 0$$

$$\Rightarrow \quad \bar{x} = A$$

Hence, mean square deviation is the minimum when $A = \bar{x}$. Consequently, SD is the minimum value of root mean square deviation.

30. Prove that SD is independent of charge in origin but not of scale. (2nd property of SD)

Solution : Let us consider, x is the original variable taking values $x_1, x_2, x_3, \ldots\ldots, x_n$, then

$$\sigma_x = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$$

After changing origin and scale the new variable becomes

$u_i = \frac{x_i - A}{h}$; A and h are constants.

$$\Rightarrow \quad x_i = A + hu_i$$

$$\Rightarrow \quad \bar{x} = A + h\bar{u}$$

$$\therefore x_i - \bar{x} = h(u_i - \bar{u})$$

Now, $\sigma_x^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{1}{n}\sum\{h(u_i - \bar{u})\}^2 = h^2\frac{1}{n}\sum(u_i - \bar{u})^2 = h^2\sigma_u^2$

$\therefore \qquad \sigma_x = h\sigma_u$

which involves h not A. Thus we have got conclusion that SD is independent of change in origin but not of scale.

31. Prove, for any discrete distribution SD is not less than MD from mean.

(3rd property of SD)

Solution : Suppose, the discrete distribution is considered as

$$x : \; x_1, x_2, x_3, \ldots\ldots, x_n$$

$$f : \; f_1, f_2, f_3, \ldots\ldots\ldots, f_n$$

Now, we have to show that

$SD < MD$ from mean

⇨ $(SD)^2 \geq (MD\ from\ mean)^2$

⇨ $\frac{1}{N}\sum f_i(x_i - \bar{x})^2 \geq \left[\frac{1}{N}\sum f_i\,|x_i - \bar{x}|\right]^2$

⇨ $\frac{1}{N}\sum f_i {z_i}^2 \geq \left[\frac{1}{N}\sum f_i\, z_i\right]^2$ putting $|x_i - \bar{x}| = z_i$

⇨ $\frac{1}{N}\sum f_i {z_i}^2 - \left[\frac{1}{N}\sum f_i\, z_i\right]^2 \geq 0$

⇨ $\frac{1}{N}\sum f_i(z_i - \bar{z})^2 \geq 0$

⇨ $\sigma_z^2 \geq 0$, which is always true. Hence the result.

32. Find the mean and standard deviation of first ***n*** natural numbers.

Solution : The first ***n*** natural numbers are 1, 2, 3, ……, n ; namely, the values of ***x*** are 1, 2, 3, ……., n.

$\therefore$ Mean = $\bar{x} = \frac{\sum x}{n} = \frac{1+2+3+\ldots\ldots+n}{n} = \frac{n(n+1)}{2} \times \frac{1}{n} = \frac{n+1}{2}$

$\therefore \quad$ SD = $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2}$

$= \sqrt{\frac{n+1}{2}\left\{\frac{2n+1}{3} - \frac{n+1}{2}\right\}}$

$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{4n+2-3n-3}{6}\right)}$

$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{n-1}{6}\right)} = \sqrt{\frac{n^2-1}{12}}$

33. If $\sum x^2 = 256$, $\bar{x} = 0$ and n = 16, find σ.

Solution : Here, we have $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{256}{16} - 0} = \sqrt{16} = 4$

34. If *SD* of x is 5, find *SD* of (i) $2x - 3$ and (ii) $\frac{x}{5} + 1$

Solution : (i) *SD* $(2x - 3)$ = 2 *SD* (x) = 2 x 5 = 10

(ii) *SD* $(\frac{x}{5} + 1) = \frac{1}{5}$ *SD* $(x) = \frac{1}{5}$ x 5 = 1

35. Find the *SD* of two values of 6 and 10.

Solution : *SD* of two values = $\frac{1}{2}$ x Difference of two values = $\frac{1}{2}$ x (10 – 6) = 2

36. Find the *SD* of the numbers 6, 6, 8, 8.

Solution :

SD of 6, 6, 8, 8 = *SD* of 6 and 8 = $\frac{1}{2}$ x Difference of two values = $\frac{1}{2}$ x (8 – 6) = 1

37. *SD* of two quantities is 5. If one of them is 12, find the other.

Solution : Suppose, other is x.

Now, we have, $\frac{1}{2}$ x $(12 - x) = 5$

⇨ $12 - x = 10$

⇨ $x = 2$ when $x < 12$

Again, $\frac{1}{2}$ x $(x - 12) = 5$

⇨ $x - 12 = 10$

⇨ $x = 22$ when $x > 12$

38. What are the relations between different measures of dispersions ?

Solution : The relations between *QD* and *SD*, *MD* and *SD*, *MD* and *QD* are given below :

1. $QD = \frac{2}{3} SD$
2. MD from mean = $\frac{4}{5} SD$
3. MD from mean = $\frac{6}{5} QD$

39. For a group of items the value of *QD* is 30. Find out the most likely value of variance.

Solution : We know that $QD = \frac{2}{3} SD$

⇨ $SD = \frac{3}{2} QD = \frac{3}{2}$ x 30 = 45 ∴ Variance = $(SD)^2$ = 2025

40. What is the probable value of MD where $Q_3 = 40$ and $Q_1 = 15$.

Solution : Here, $QD = \frac{Q_3 - Q_1}{2} = \frac{40-15}{2} = 12.5$

We know that $MD = \frac{6}{5} QD$

⇨ $MD = \frac{6 \text{ x } 12.5}{5} = 15$

41. Write the merits and demerits of SD.

Solution : **Merits :**

It is the best measure of dispersion because of the following merits :

(i) It is based on all observations.

(ii)n It is suitable for further mathematical treatment.

(iii) It is less affected by fluctuations of sampling than other measures of dispersion.

(iv) It is rigidly defined.

(v) It has wide applications in statistical theory.

Demerits :

(i) It is difficult to calculate.

(ii) It is affected by extreme values.

42. Write the uses of different measures of dispersion.

Solution :

(i) **Range :** Range is widely used in Statistical Quality Control (S.Q.C.), Weather Forecasting, Stock Market Fluctuations, etc.

(ii) **Quartile Deviation :** QD is used in elementary descriptive statistics where the distribution is open end. In case of attributes, QD is more appropriate.

(iii) **Mean Deviation :** MD is frequently used by the economic and business statisticians for its simplicity. It is used in computing the distribution of wealth in a community.

(iv) **Standard Deviation :** SD is used in statistical theory such as skewness, kurtosis, correlation and regression analysis, sampling theory and tests of significance etc. It is most widely used measure of dispersion.

43. What is Co-efficient of Variation ?

Solution : To compare the variability of two series, the relative measure of dispersion based on standard deviation is called the co-efficient of standard deviation.

Co-efficient of SD = $\frac{\sigma}{\bar{x}}$

The co-efficient of SD multiplied by 100 gives the co-efficient of variation. This was introduced by Karl Pearson.

Co-efficient of Variation (CV) = $\frac{\sigma}{\bar{x}}$ x 100

It indicates the relationship between the SD and the AM expressed in percentage. This is a pure number independent of units.

44. If $n = 5, \bar{x} = 4, \sum x^2 = 90$, find the co-efficient of variation.

Solution : Here, we have,

$$\sigma^2 = \frac{1}{n}\sum x^2 - \bar{x}^2 = \frac{1}{5} \text{ x } 90 - 16 = 18 - 16 = 2$$

⇨ $\sigma = 1.41$,

∴ CV = $\frac{\sigma}{\bar{x}}$ x 100 = $\frac{1.41 \text{ x } 100}{4}$ = 35.25%

45. For a distribution, the co-efficient of variation is 35.3% and the value of arithmetic mean is 4. Find the value of SD.

Solution : Here, we have,

CV = $\frac{\sigma}{\bar{x}}$ x 100

⇨ $\sigma = \frac{CV \text{ x } \bar{x}}{100} = \frac{35.3 \text{ x } 4}{100}$ = 1.412

46. Co-efficient of two series are 60% and 80%. Their standard deviations are 20 and 16. What are their arithmetic means ?

Solution : Here, we have,

For the first series, CV = $\frac{\sigma}{\bar{x}}$ x 100

⇨ $\bar{x} = \frac{\sigma}{CV}$ x 100

⇨ $\bar{x} = \frac{20}{60}$ x 100

⇨ $\bar{x}$ = 33.3

For the second series, CV = $\frac{\sigma}{\bar{x}}$ x 100

⇨ $\bar{x} = \frac{\sigma}{CV}$ x 100

⇨ $\bar{x} = \frac{16}{80}$ x 100

⇨ $\bar{x}$ = 20

47. Find the co-efficient of variation from the following data :

Wages :	12-13	13-14	14-15	15-16	16-17	17-18	18-19
No. of persons :	15	30	44	60	30	14	7

Solution :

wages	f	x	$d = x - 15.5$	fd	fd^2
12-13	15	12.5	–3	–45	135
13-14	30	13.5	–2	–60	120
14-15	44	14.5	–1	–44	44
15-16	60	15.5	0	0	0
16-17	30	16.5	1	30	30
17-18	14	17.5	2	28	56
18-19	7	18.5	3	21	63
	N = 200			$\sum fd = -70$	$\sum fd^2 = 448$

$$\sigma = \sqrt{\frac{1}{N}\sum fd^2 - \left(\frac{1}{N}\sum fd\right)^2}$$

$$= \sqrt{\frac{448}{200} - \left(\frac{-70}{200}\right)^2}$$

$$= \sqrt{2.24 - 0.1225} = 1.46$$

$$\bar{x} = A + \frac{\sum fd}{N}$$

$$= 15.5 + \frac{-70}{200}$$

$$= 15.5 - 0.35$$

$$= 15.15$$

$$\text{CV} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.46}{15.15} \times 100$$

$$= 9.64\%$$