Exercise 2(D)

1. $(\boldsymbol{x^2+y^2})\boldsymbol{p}+\boldsymbol{2xyq}=\boldsymbol{z(x+y)}$

Solution : Given PDE is

$(x^2+y^2)p+2xyq=z(x+y)$ .... (i)

The Lagrange's auxiliary equations for (i) are,

$\frac{dx}{x^2+y^2}=\frac{dy}{2xy}=\frac{dz}{z(x+y)}$ .... (ii)

Choosing 1,1,0 as multipliers, each fraction for (ii)

$=\frac{dx+dy}{x^2+y^2+2xy}=\frac{d(x+y)}{(x+y)^2}$ .... (iii)

From 3rd fraction of (ii) and fraction (iii), we get

$\frac{dz}{z(x+y)}=\frac{d(x+y)}{(x+y)^2}$

$\Rightarrow \frac{dz}{z}=\frac{d(x+y)}{(x+y)}$

$\Rightarrow \int\frac{dz}{z}+logc_1=\int\frac{d(x+y)}{(x+y)}$, where c_1 is integrating constant.

$\Rightarrow logz+logc_1=\log(x+y)$

$\Rightarrow logc_1=\log(x+y)-logz$

$\therefore\ c_1=\frac{x+y}{z}$ .... (iv)

Choosing $1,-1,0$ as multipliers, each fraction for (ii)

$=\frac{dx-dy}{x^2+y^2-2xy}=\frac{d(x-y)}{(x-y)^2}$ .... (v)

From fraction (iii) & fraction (v), we get

$$\frac{d(x+y)}{(x+y)^2}=\frac{d(x-y)}{(x-y)^2}$$

$\Rightarrow \int \frac{d(x+y)}{(x+y)^2} = \int \frac{d(x-y)}{(x-y)^2} + c_2$, where c_2 is integrating constant.

$\Rightarrow \quad -\frac{1}{x+y} = -\frac{1}{x-y} + c_2$

$\Rightarrow \frac{1}{x-y} - \frac{1}{x+y} = c_2$

$\therefore \ c_2 = \frac{2y}{x^2-y^2}$ .... (vi)

From (iv) & (vi), the general solution is

$\varphi(c_1, c_2) = 0$

$i.e., \varphi\left(\frac{x+y}{z}, \frac{2y}{x^2-y^2}\right) = 0$, where φ is an arbitrary function. **Answer**

2. $\{\boldsymbol{y(x+y)+az}\}\boldsymbol{p} + \{\boldsymbol{x(x+y)-az}\}\boldsymbol{q} = \boldsymbol{z(x+y)}$

$Solution:$ Given PDE is

$\{y(x+y)+az\}p + \{x(x+y)-az\}q = z(x+y)$ .... (i)

The Lagrange's auxiliary equations for (i) are,

$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ .... (ii)

Choosing 1,1,0 as multipliers, each fraction for (ii)

$= \frac{dx+dy}{y(x+y)+az+x(x+y)-az} = \frac{d(x+y)}{(x+y)^2}$ .... (iii)

From 3rd fraction of (ii) and fraction (iii), we get

$\frac{dz}{z(x+y)} = \frac{d(x+y)}{(x+y)^2}$

$\Rightarrow \frac{dz}{z} = \frac{d(x+y)}{(x+y)}$

$\Rightarrow \int \frac{dz}{z} + log c_1 = \int \frac{d(x+y)}{(x+y)}$, where c_1 is integrating constant.

$\Rightarrow logz + logc_1 = \log(x+y)$

$\Rightarrow logc_1 = \log(x+y) - logz$

$\therefore\ c_1 = \frac{x+y}{z}$ .... (iv)

Choosing $-x, y, 0$ as multipliers, each fraction for (ii)

$= \frac{-xdx+ydy}{-xy(x+y)-axz+xy(x+y)-ayz} \quad = \frac{-xdx+ydy}{-az(x+y)}$ .... (v)

From 3rd fraction of (ii) & fraction (v), we get

$$\frac{dz}{z(x+y)} = \frac{-xdx+ydy}{-az(x+y)}$$

$\Rightarrow -adz = -xdx + ydy$

$\Rightarrow -a\int dz = -\int xdx + \int ydy + \frac{c_2}{2}$, where c_2 is integrating constant.

$\Rightarrow\ \ -az = -\frac{x^2}{2} + \frac{y^2}{2} + \frac{c_2}{2}$

$\Rightarrow -2az + x^2 - y^2 = c_2$

$\therefore\quad c_2 = x^2 - y^2 - 2az$ .... (vi)

From (iv) & (vi), the general solution is

$\varphi(c_1, c_2) = 0$

$i.e., \varphi\left(\frac{x+y}{z}, x^2 - y^2 - 2az\right) = 0$, where φ is an arbitrary function. **Answer**

Or, $\frac{x+y}{z} = \varphi(x^2 - y^2 - 2az)$ **Answer**

3. $(\boldsymbol{y^2 + yz + z^2})\boldsymbol{p} + (\boldsymbol{z^2 + zx + x^2})\boldsymbol{q} = (\boldsymbol{x^2 + xy + y^2})$

Solution : Given equation is,

$(y^2 + yz + z^2)p + (z^2 + zx + x^2)q = (x^2 + xy + y^2)$ .... (i)

The Lagrange's auxiliary equations for (i) are,

$$\frac{dx}{y^2+yz+z^2} = \frac{dy}{z^2+zx+x^2} = \frac{dz}{x^2+xy+y^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Choosing $1, -1, 0$ as multipliers, each fraction for (ii)

$$= \frac{dx-dy}{(y^2+yz+z^2)-(z^2+zx+x^2)} = \frac{d(x-y)}{(y^2-x^2)+z(y-x)} = \frac{d(x-y)}{(y-x)(x+y+z)} \ldots\ldots\text{(iii)}$$

Choosing $0, 1, -1$ as multipliers, each fraction for (ii)

$$= \frac{dy-dz}{(z^2+zx+x^2)-(x^2+xy+y^2)} = \frac{d(y-z)}{(z^2-y^2)+x(z-y)} = \frac{d(y-z)}{(z-y)(x+y+z)} \ldots\ldots\text{(iv)}$$

Choosing $-1, 0, 1$ as multipliers, each fraction for (ii)

$$= \frac{dz-dx}{(x^2+xy+y^2)-(y^2+yz+z^2)} = \frac{d(z-x)}{(x^2-z^2)+y(x-z)} = \frac{d(z-x)}{(x-z)(x+y+z)} \ldots\ldots\text{(v)}$$

From fraction (iii) & fraction (iv), we get

$$\frac{d(x-y)}{(y-x)(x+y+z)} = \frac{d(y-z)}{(z-y)(x+y+z)}$$

$$\Rightarrow \frac{d(x-y)}{x-y} = \frac{d(y-z)}{y-z}$$

$\Rightarrow \int \frac{d(x-y)}{x-y} = \int \frac{d(y-z)}{y-z} + log c_1$, where c_1 is an integrating constant.

$$\Rightarrow log(x-y) = log(y-z) + log c_1$$

$$\Rightarrow \frac{x-y}{y-z} = c_1 \ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

From fraction (iv) & fraction (v), we get

$$\frac{d(y-z)}{(z-y)(x+y+z)} = \frac{d(z-x)}{(x-z)(x+y+z)}$$

$$\Rightarrow \frac{d(y-z)}{y-z} = \frac{d(z-x)}{z-x}$$

$\Rightarrow \int \frac{d(y-z)}{y-z} = \int \frac{d(z-x)}{z-x} + log c_2$, where c_2 is an integrating constant.

$\Rightarrow log(y - z) = log(z - x) + \ logc_2$

$\Rightarrow \frac{y-z}{z-x} = c_2$ ……………….. (vii)

From (vi) & (vii), the general solution is

$\varphi(c_1, c_2) = 0$

$i.e., \varphi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0$, where φ is an arbitrary function. **Answer**