Exercise 5.5

9. $x^{sinx} + (sinx)^{cosx}$

Solution: ধৰাহ'ল, $y = x^{sinx} + (sinx)^{cosx}$ …….. (i)

ধৰাহ'ল, $p = x^{sinx}$ আৰু $q = (sinx)^{cosx}$

$\therefore \log p = sinx \log x$

⇨ $\frac{d}{dx}(logp) = \frac{d}{dx}\{sinx \log x\}$

⇨ $\frac{1}{p}.\frac{dp}{dx} = cosx.logx + sinx.\frac{1}{x} = \frac{x.cosx.logx+sinx}{x}$

⇨ $\frac{dp}{dx} = p \times \frac{x.cosx.logx+sinx}{x}$

⇨ $\frac{dp}{dx} = x^{sinx}.\frac{x.cosx.logx+sinx}{x}$

$\therefore \frac{dp}{dx} = x^{sinx-1}(x.cosx.logx + sinx)$ ……(ii)

দ্বিতীয়তে, $logq = log(sinx)^{cosx}$

⇨ $logq = cosx \log(sinx)$

⇨ $\frac{d}{dx}(logq) = \frac{d}{dx}\{cosx \log(sinx)\}$

⇨ $\frac{1}{q}.\frac{dq}{dx} = (-sinx)\log(sinx) + cosx.\frac{1}{sinx}.cosx$

⇨ $\frac{dq}{dx} = q \times \left[\frac{-sin^2x.\log(sinx)+\cos^2 x}{sinx}\right]$

⇨ $\frac{dq}{dx} = (sinx)^{cosx}\left[\frac{-sin^2x.\log(sinx)+\cos^2 x}{sinx}\right]$

$\therefore \frac{dq}{dx} = (sinx)^{cosx-1}[-sin^2x.\log(sinx) + \cos^2 x]$ ……. (iii)

(i) ৰ পৰা আমি পাওঁ,

$$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$$ [(ii)আৰু (iii)ৰ সহায়ত]

$\therefore \frac{dy}{dx} = x^{sinx-1}(x.cosx.logx + sinx) + (sinx)^{cosx-1}[-sin^2x.\log(sinx) + \cos^2 x]$ **Answer**

10. $x^{xcosx} + \frac{x^2+1}{x^2-1}$

Solution: ধৰাহ'ল, $y = x^{xcosx} + \frac{x^2+1}{x^2-1}$ …….. (i)

ধৰাহ'ল, $p = x^{xcosx}$ আৰু $q = \frac{x^2+1}{x^2-1}$

$\therefore \log p = xcosx \log x$

⇨ $\frac{d}{dx}(logp) = \frac{d}{dx}\{xcosx \log x\}$

⇨ $\frac{1}{p}.\frac{dp}{dx} = xcosx.\frac{1}{x} + (cosx - xsinx)logx$

⇨ $\frac{1}{p}.\frac{dp}{dx} = cosx + (cosx - xsinx)logx$

⇨ $\frac{dp}{dx} = p[cosx + (cosx - xsinx)logx]$

$\therefore \frac{dp}{dx} = x^{xcosx}[cosx + (cosx - xsinx)logx]$ ……..(ii)

দ্বিতীয়তে, $q = \frac{x^2+1}{x^2-1}$

⇨ $\frac{dq}{dx} = \frac{(x^2-1).2x-(x^2+1).2x}{(x^2-1)^2}$

$\therefore \frac{dq}{dx} = \frac{-4x}{(x^2-1)^2}$ ……...(iii)

(i) ৰ পৰা আমি পাওঁ,

$$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$$ [(ii)আৰু (iii)ৰ সহায়ত]

$$\therefore \frac{dy}{dx} = x^{xcosx}[cosx + (cosx - xsinx)logx] - \frac{-4x}{(x^2-1)^2}$$ **Answer**

11. $(xcosx)^x + (xsinx)^{\frac{1}{x}}$

Solution: ধৰাহ'ল, $y = (xcosx)^x + (xsinx)^{\frac{1}{x}}$ …….. (i)

ধৰাহ'ল, $p = (xcosx)^x$ আৰু $q = (xsinx)^{\frac{1}{x}}$

$\therefore \log p = x\log(xcosx)$

⇨ $\frac{d}{dx}(logp) = \frac{d}{dx}\{x\log(xcosx)\}$

⇨ $\frac{1}{p}.\frac{dp}{dx} = \log(xcosx) + x.\frac{1}{xcosx}.(cosx - xsinx)$

⇨ $\frac{dp}{dx} = p\left[\log(xcosx) + \frac{1}{cosx}.(cosx - xsinx)\right]$

$\therefore \frac{dp}{dx} = (xcosx)^x[\log(xcosx) + 1 - xtanx]$ ……(ii)

দ্বিতীয়তে, $logq = log(xsinx)^{\frac{1}{x}}$

⇨ $logq = \frac{1}{x}log(xsinx)$

⇨ $\frac{d}{dx}(logq) = \frac{d}{dx}\left\{\frac{1}{x}log(xsinx)\right\}$

⇨ $\frac{1}{q}.\frac{dq}{dx} = -\frac{1}{x^2}.\log(xsinx) + \frac{1}{x}.\frac{1}{xsinx}.(sinx + xcosx)$

⇨ $\frac{dq}{dx} = q\left[-\frac{1}{x^2}.\log(xsinx) + \frac{1}{x^2} + \frac{cotx}{x}\right]$

$$\therefore \frac{dq}{dx} = (xsinx)^{\frac{1}{x}}\left[-\frac{1}{x^2}.\log(xsinx) + \frac{1}{x^2} + \frac{cotx}{x}\right]\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

(i) ৰ পৰা আমি পাওঁ,

$$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$$ [(ii)আৰু (iii)ৰ সহায়ত]

$$\therefore \frac{dy}{dx} = (xcosx)^x[\log(xcosx) + 1 - xtanx] + (xsinx)^{\frac{1}{x}}\left[-\frac{1}{x^2}.\log(xsinx) + \frac{1}{x^2} + \frac{cotx}{x}\right]$$ **Ans.**