<u>Exercise 5.5</u>

5. $(x+3)^2.(x+4)^3.(x+5)^4$

Solution: ধৰাহ’ল, $y=(x+3)^2.(x+4)^3.(x+5)^4$ ………….. (i)

(i) ৰ দুয়োপক্ষত log ব্যৱহাৰ কৰি আমি পাওঁ,

$$\log y=\log\{(x+3)^2.(x+4)^3.(x+5)^4\}$$

⇨ $\log y=\log(x+3)^2+\log(x+4)^3+\log(x+5)^4$

⇨ $\log y=2\log(x+3)+3\log(x+4)+4\log(x+5)$

⇨ $\frac{d}{dx}(\log y)=\frac{d}{dx}[2\log(x+3)+3\log(x+4)+4\log(x+5)]$

⇨ $\frac{1}{y}.\frac{dy}{dx}=2.\frac{1}{x+3}+3.\frac{1}{x+4}+4.\frac{1}{x+5}$

⇨ $\frac{dy}{dx}=y\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$

$\therefore$ $\frac{dy}{dx}=(x+3)^2.(x+4)^3.(x+5)^4\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$ **Answer**

6. $\left(x+\frac{1}{x}\right)^x+x^{\left(1+\frac{1}{x}\right)}$

Solution: ধৰাহ’ল, $y=\left(x+\frac{1}{x}\right)^x+x^{\left(1+\frac{1}{x}\right)}$ ……………........(i)

ধৰাহ’ল, $p=\left(x+\frac{1}{x}\right)^x$ আৰু $q=x^{\left(1+\frac{1}{x}\right)}$

$\therefore$ $\log p=x\log\left(x+\frac{1}{x}\right)$

⇨ $\frac{d}{dx}(logp)=\frac{d}{dx}\left\{x\log\left(x+\frac{1}{x}\right)\right\}$

⇨ $\frac{1}{p}.\frac{dp}{dx}=log\left(x+\frac{1}{x}\right)+x\times\frac{1}{\left(x+\frac{1}{x}\right)}\times\left(1-\frac{1}{x^2}\right)$

$\Rightarrow \frac{1}{p}.\frac{dp}{dx} = log\left(x+\frac{1}{x}\right) + \frac{x^2}{x^2+1} \times \frac{x^2-1}{x^2}$

$\Rightarrow \frac{1}{p}.\frac{dp}{dx} = log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}$

$\Rightarrow \frac{dp}{dx} = p\left[log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right]$

$\therefore \frac{dp}{dx} = \left(x+\frac{1}{x}\right)^x \left[log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right]$ ....(ii)

দ্বিতীয়তে, $logq = logx^{\left(1+\frac{1}{x}\right)}$

$\Rightarrow logq = \left(1+\frac{1}{x}\right)\log x$

$\Rightarrow \frac{d}{dx}(logq) = \frac{d}{dx}\left\{\left(1+\frac{1}{x}\right)\log x\right\}$

$\Rightarrow \frac{1}{q}.\frac{dq}{dx} = \left(-\frac{1}{x^2}\right)logx + \left(1+\frac{1}{x}\right).\frac{1}{x}$

$\Rightarrow \frac{dq}{dx} = q\left[-\frac{logx}{x^2} + \frac{x+1}{x^2}\right]$

$\therefore \frac{dq}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[\frac{-logx+x+1}{x^2}\right]$ .... (iii)

(i) ৰ পৰা আমি পাওঁ,

$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$ [(ii)আৰু (iii)ৰ সহায়ত]

$\therefore \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{-logx+x+1}{x^2}\right]$ **Answer**

7. $(\log x)^x + x^{logx}$

Solution: ধৰাহ'ল, $y = (\log x)^x + x^{logx}$ ....(i)

ধৰাহ'ল, $p = (\log x)^x$ আৰু $q = x^{logx}$

$\therefore\ \log p = \log(\log x)^x = x\log(\log x)$

$\Rightarrow \frac{d}{dx}(logp) = \frac{d}{dx}\{x\log(\log x)\}$

$\Rightarrow \frac{1}{p}.\frac{dp}{dx} = \log(\log x) + x.\frac{1}{\log x}.\frac{1}{x}$

$\Rightarrow \frac{dp}{dx} = p\left[\log(\log x) + \frac{1}{\log x}\right]$

$\therefore\ \frac{dp}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$ ....(ii)

দ্বিতীয়তে, $logq = logx^{logx} = \log x.\log x = (\log x)^2$

$\Rightarrow \frac{d}{dx}(logq) = \frac{d}{dx}(\log x)^2$

$\Rightarrow \frac{1}{q}.\frac{dq}{dx} = 2.\log x.\frac{1}{x} = \frac{2}{x}\log x$

$\Rightarrow \frac{dq}{dx} = q.\frac{2}{x}\log x = x^{logx}.\frac{2\log x}{x}$

$\Rightarrow \frac{dq}{dx} = 2\log x.\frac{x^{logx}}{x} = 2\log x.x^{logx-1}$

$\therefore\ \frac{dq}{dx} = 2x^{logx-1}\log x$ ....(iii)

(i) ৰ পৰা আমি পাওঁ,

$$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$$

$\therefore\ \frac{dy}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right] + 2x^{logx-1}\log x$ [(ii)আৰু (iii)ৰ সহায়ত] **Answer**

8. $(sinx)^x + \sin^{-1}\sqrt{x}$

Solution: ধৰাহ'ল, $y = (sinx)^x + \sin^{-1}\sqrt{x}$ ....(i)

ধৰাহ'ল, $p = (sinx)^x$ আৰু $q = \sin^{-1}\sqrt{x}$

$\therefore \log p = \log(sinx)^x = x\log(sinx)$

$\Rightarrow \frac{d}{dx}(logp) = \frac{d}{dx}\{x\log(sinx)\}$

$\Rightarrow \frac{1}{p}.\frac{dp}{dx} = \log(sinx) + x.\frac{1}{sinx}.cosx$

$\Rightarrow \frac{dp}{dx} = p[\log(sinx) + xcotx]$

$\therefore \frac{dp}{dx} = (sinx)^x[\log(sinx) + xcotx]$ ………………...(ii)

দ্বিতীয়তে, $q = \sin^{-1}\sqrt{x}$

$\Rightarrow \frac{dq}{dx} = \frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)$

$\Rightarrow \frac{dq}{dx} = \frac{1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}}$ $\left[\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \ \& \ \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\right]$

$\therefore \frac{dq}{dx} = \frac{1}{2.\sqrt{x(1-x)}}$ ……………………..(iii)

(i) ৰ পৰা আমি পাওঁ,

$\frac{dy}{dx} = \frac{dp}{dx} + \frac{dq}{dx}$

$\therefore \frac{dy}{dx} = (sinx)^x[\log(sinx) + xcotx] + \frac{1}{2.\sqrt{x(1-x)}}$ [(ii)আৰু (iii)ৰ সহায়ত] ↓**Answer**