Exercise 5.5

Differentiate the functions given in Exercise 1 to 11 w.r.t.x

(1 ৰ পৰা 11 লৈকে ফলনসমূহ x ৰ সাপেক্ষে অৱকলজ কৰা) :

1. $cosx.cos2x.cos3x$

Solution: ধৰাহ'ল, $y = cosx.cos2x.cos3x$ …….. (i)

(i) ৰ দুয়োপক্ষত log ব্যৱহাৰ কৰি আমি পাওঁ,

$logy = log(cosx.cos2x.cos3x)$

$\Rightarrow logy = \log(cosx) + \log(cos2x) + \log(cos3x)$

$\Rightarrow \frac{d}{dx}(logy) = \frac{d}{dx}[\log(cosx) + \log(cos2x) + \log(cos3x)]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{cosx}(-sinx) + \frac{1}{cos2x}(-sin2x).2 + \frac{1}{cos3x}(-sin3x).3$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{sinx}{cosx} - \frac{2.sin2x}{cos2x} - \frac{3.sin3x}{cos3x} = -\tan x - 2\tan 2x - 3\tan 3x$

$\Rightarrow \frac{dy}{dx} = -y \times [\tan x + 2\tan 2x + 3\tan 3x]$

$\therefore \frac{dy}{dx} = -(cosx.cos2x.cos3x) \times [tanx + 2tan2x + 3tan3x]$ **Answer**

2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Solution: ধৰাহ'ল, $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ …….. (i)

(i) ৰ দুয়োপক্ষক বর্গ কৰি আমি পাওঁ,

$y^2 = \frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}$ ……..(ii)

(ii) ৰ দুয়োপক্ষত log ব্যৱহাৰ কৰি আমি পাওঁ,

$$2logy = log(x-1) + log(x-2) - log(x-3) - log(x-4) - log(x-5)$$

$$\frac{d}{dx}(2logy) = \frac{d}{dx}[log(x-1) + log(x-2) - log(x-3) - log(x-4) - log(x-5)]$$

$$\Rightarrow \frac{2}{y}\frac{dy}{dx} = \frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$ **Answer**

3. $(logx)^{cosx}$

Solution: ধৰাহ’ল, $y = (logx)^{cosx}$ ..... (i)

(i) ৰ দুয়োপক্ষত log ব্যৱহাৰ কৰি আমি পাওঁ,

$$logy = log[(logx)^{cosx}]$$

$$\Rightarrow logy = cosx.log(logx)$$
$$\Rightarrow \frac{d}{dx}(logy) = \frac{d}{dx}\{cosx.log(logx)\}$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -sinx.log(logx) + cosx.\frac{1}{logx}.\frac{1}{x}$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -sinx.log(logx) + \frac{cosx}{xlogx}$$
$$\Rightarrow \frac{dy}{dx} = y.\left\{-sinx.log(logx) + \frac{cosx}{xlogx}\right\}$$

$$\therefore \frac{dy}{dx} = (logx)^{cosx}.\left\{-sinx.log(logx) + \frac{cosx}{xlogx}\right\}$$ **Answer**

4. $x^x - 2^{sinx}$

Solution: ধৰাহ’ল, $y = x^x - 2^{sinx}$ ..... (i)

ধৰাহ’ল, $p = x^x$ আৰু $q = 2^{sinx}$

$\therefore logp = xlogx$ $[\because logx^x = xlogx]$

$\Rightarrow \frac{d}{dx}(logp) = \frac{d}{dx}(xlogx)$

$\Rightarrow \frac{1}{p}.\frac{dp}{dx} = logx + x.\frac{1}{x}$

$\Rightarrow \frac{dp}{dx} = p(logx + 1)$

$\Rightarrow \frac{dp}{dx} = x^x(logx + 1)$ …….. (ii)

দ্বিতীয়তে, $logq = log2^{sinx}$

$\Rightarrow logq = sinx.log2$

$\Rightarrow \frac{d}{dx}(logq) = \frac{d}{dx}(sinx.log2)$

$\Rightarrow \frac{1}{q}.\frac{dq}{dx} = log2.\frac{d}{dx}(sinx) = log2.cosx$

$\Rightarrow \frac{dq}{dx} = q.log2.cosx$

$\Rightarrow \frac{dq}{dx} = log2^{sinx}.log2.cosx = log2.log2^{sinx}.cosx$

(i) ৰ পৰা আমি পাওঁ,

$$\frac{dy}{dx} = \frac{dp}{dx} - \frac{dq}{dx}$$

$\therefore \frac{dy}{dx} = x^x(logx + 1) - log2.log2^{sinx}.cosx$ **Answer**

Exercise 5.4 ৰ ৪ নং প্ৰশ্নটো শুদ্ধ কৰি লবা

8. $\log(logx),\ x > 1$

Solution : ধৰাহ'ল, y = $\log(logx)$ ← হব লাগে ।