Exercise 5.4

6. $e^x + e^{x^2} + \cdots + e^{x^5}$

Solution : ধৰাহ'ল, $y = e^x + e^{x^2} + \cdots + e^{x^5}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{x^2} + \cdots + e^{x^5}\right)$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$

$= e^x + 2x.e^{x^2} + 3x^2.e^{x^3} + 4x^3.e^{x^4} + 5x^5.e^{x^5}$ $\quad \left[\because \frac{d}{dx}'\left(e^{x^n}\right) = e^{x^n}.\frac{d}{dx}\,x^n = nx^{n-1}.e^{x^n}\right]$

Answer

7. $\sqrt{e^{\sqrt{x}}},\ x > 0$

Solution : ধৰাহ'ল, $y = \sqrt{e^{\sqrt{x}}}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right)$

$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}.\frac{d}{dx}\left(e^{\sqrt{x}}\right)$ $\quad \left[\because \frac{d}{dx}'\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\right]$

$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}.e^{\sqrt{x}}.\frac{d}{dx}\left(\sqrt{x}\right)$ $\quad \left[\because \frac{d}{dx}'\left(e^{x^n}\right) = e^{x^n}.\frac{d}{dx}\,x^n = nx^{n-1}.e^{x^n}\right]$

$= \frac{e^{\sqrt{x}}}{2\sqrt{e^{\sqrt{x}}}}.\frac{1}{2\sqrt{x}}$ $\quad \left[\because \sqrt{a}\,.\sqrt{b} = \sqrt{ab}\right]$

$= \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$ Answer

8. $\log(logx),\ x > 1$

Solution : ধৰাহ'ল, $y = \sqrt{e^{\sqrt{x}}}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\{\log(logx)\}$

$= \frac{1}{logx}.\frac{d}{dx}(logx)$ $\quad \left[\because \frac{d}{dx}'(logx) = \frac{1}{x}\right]$

$= \frac{1}{xlogx}$ Answer

9. $\frac{cosx}{logx}$, $x > 0$

Solution : ধৰাহ'ল, $y = \frac{cosx}{logx}$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{cosx}{logx}\right)$

$= \frac{logx.\frac{d}{dx}(cosx) - cosx.\frac{d}{dx}(logx)}{(logx)^2}$ $\left[\because \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}\right]$

$= \frac{logx.(-sinx) - cosx.\frac{1}{x}}{(logx)^2}$

$= \frac{-x.sinx.logx - cosx}{x(logx)^2}$ Answer

10. $cos(logx + e^x)$

Solution : ধৰাহ'ল, $y = cos(logx + e^x)$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\{cos(logx + e^x)\}$

$= -\{sin(logx + e^x)\}.\frac{d}{dx}(logx + e^x)$

$= -\{sin(logx + e^x)\}\left(\frac{1}{x} + e^x\right)$

$= -\{sin(logx + e^x)\}.\frac{1+xe^x}{x}$

$= \frac{-(1+xe^x)\{sin(logx+e^x)\}}{x}$ Answer

Remember a few important formulae on Logarithms (for next exercise) :

1. $\log_a p = \frac{\log_b p}{\log_b a}$
2. $\log_b pq = \log_b p + \log_b q$
3. $\log_b p^2 = 2\log_b p = \log_b p + \log_b p$
4. $\log_b p^n = n\log_b p$
5. $\log_b \frac{x}{y} = \log_b x - \log_b y$