Exercise 5.3

9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $\quad [\because \sin^{-1}(sinx) = x,\ \cos^{-1}(cosx) = x]$

Solution : দিয়া আছে,

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ .....(i)

ধৰাহ' ল, $x = tan\theta$

$\therefore \quad \frac{2x}{1+x^2} = \frac{2tan\theta}{1+tan^2\theta} = sin2\theta$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$y = \sin^{-1}(sin2\theta)$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\tan^{-1} x \qquad [\because x = tan\theta \Rightarrow \theta = \tan^{-1} x\ ]$

$\Rightarrow \frac{dy}{dx} = 2.\frac{d}{dx}(\tan^{-1} x)$

$\Rightarrow \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$ $\quad$ Answer

10. $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Solution : দিয়া আছে,

$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ .....(i)

ধৰাহ' ল, $x = tan\theta$

$\therefore \quad \frac{3x-x^3}{1-3x^2} = \frac{3tan\theta-\tan^3\theta}{1-3tan^2\theta} = tan3\theta$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$y = \tan^{-1}(tan3\theta)$

$\Rightarrow y = 3\theta$

$\Rightarrow y = 3\tan^{-1} x \qquad [\because x = tan\theta \Rightarrow \theta = \tan^{-1} x\ ]$

$\Rightarrow \frac{dy}{dx} = 3.\frac{d}{dx}(\tan^{-1} x)$

$\Rightarrow \frac{dy}{dx} = 3.\frac{1}{1+x^2} = \frac{3}{1+x^2}$ Answer

11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad 0 < x < 1$

Solution : দিয়া আছে,

$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ .....(i)

ধৰাহ' ল, $x = tan\theta$

$\therefore \quad \frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+tan^2\theta} = cos2\theta$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$y = \cos^{-1}(cos2\theta)$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\tan^{-1} x$ $[\because x = tan\theta \Rightarrow \theta = \tan^{-1} x]$

$\Rightarrow \frac{dy}{dx} = 2.\frac{d}{dx}(\tan^{-1} x)$

$\Rightarrow \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$ Answer

12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad 0 < x < 1$

Solution : দিয়া আছে,

$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ .....(i)

ধৰাহ' ল, $x = tan\theta$

$\therefore \quad \frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+tan^2\theta} = cos2\theta = sin\left(\frac{\pi}{2} - 2\theta\right)$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$$y = \sin^{-1}\left\{sin\left(\frac{\pi}{2} - 2\theta\right)\right\}$$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - 2.\frac{d}{dx}(\tan^{-1} x)$ $\quad [\because x = tan\theta \Rightarrow \theta = \tan^{-1} x\ ]$

$\Rightarrow \frac{dy}{dx} = 0 - 2.\frac{1}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$ $\quad$ Answer

13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), \quad -1 < x < 1$

Solution : দিয়া আছে,

$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ .....(i)

ধৰাহ' ল, $x = tan\theta$

$\therefore \quad \frac{2x}{1+x^2} = \frac{2tan\theta}{1+tan^2\theta} = sin2\theta = cos\left(\frac{\pi}{2} - 2\theta\right)$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$$y = \cos^{-1}\left\{cos\left(\frac{\pi}{2} - 2\theta\right)\right\}$$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - 2.\frac{d}{dx}(\tan^{-1} x)$ $\quad [\because x = tan\theta \Rightarrow \theta = \tan^{-1} x\ ]$

$\Rightarrow \frac{dy}{dx} = 0 - 2.\frac{1}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$ $\quad$ Answer

14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Solution : দিয়া আছে,

$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ .....(i)

ধৰাহ' ল, $x = sin\theta$

$\therefore \quad 2x\sqrt{1-x^2} = 2sin\theta.\sqrt{1-\sin^2\theta} = 2sin\theta cos\theta = sin2\theta$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$y = \sin^{-1}(sin2\theta)$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\sin^{-1} x \qquad [\because x = sin\theta \Rightarrow \theta = \sin^{-1} x]$

$\Rightarrow \frac{dy}{dx} = 2.\frac{d}{dx}(\sin^{-1} x)$

$\Rightarrow \frac{dy}{dx} = 2.\frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$ Answer

15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), \quad 0 < x < \frac{1}{\sqrt{2}}$

Solution : দিয়া আছে,

$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ .....(i)

ধৰাহ' ল, $x = cos\theta$

$\therefore \quad \frac{1}{2x^2-1} = \frac{1}{2cos^2\theta-1} = \frac{1}{cos2\theta} = sec2\theta$

$\therefore$ (i) ৰ পৰা আমি পাওঁ,

$y = \sec^{-1}(sec2\theta)$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\cos^{-1} x \qquad [\because x = cos\theta \Rightarrow \theta = \cos^{-1} x]$

$\Rightarrow \frac{dy}{dx} = 2.\frac{d}{dx}(\cos^{-1} x)$

$\Rightarrow \frac{dy}{dx} = 2.\frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$ Answer