Formulae for Differentiation (অৱকলজৰ সূত্ৰ) **:**

Standard Derivatives :

(1) $\frac{d}{dx}(x^n) = nx^{n-1}$

(2) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

(3) $\frac{d}{dx}(x) = 1$

(4) $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

(5) $\frac{d}{dx}(e^x) = e^x$

(6) $\frac{d}{dx}(a^x) = a^x \log_e a$

(7) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(8) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$

(9) $\frac{d}{dx}(sinx) = cosx$

(10) $\frac{d}{dx}(cosx) = -sinx$

(11) $\frac{d}{dx}(tanx) = sec^2x$

(12) $\frac{d}{dx}(cotx) = -cosec^2x$

(13) $\frac{d}{dx}(secx) = secx.tanx$

(14) $\frac{d}{dx}(cosecx) = -cosecx.cotx$

(15) $\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(16) $\frac{d}{dx}(cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

(17) $\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$

(18) $\frac{d}{dx}(cot^{-1}x) = -\frac{1}{1+x^2}$

(19) $\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(20) $\frac{d}{dx}(cosec^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$

Exercise 5.3

Find $\frac{dy}{dx}$ in the following (তলৰ ফলনসমূহৰ $\frac{dy}{dx}$ উলিওৱা) :

1. $2x + 3y = sinx$

Solution : দিয়া আছে,

$$2x + 3y = sinx$$

$$\Rightarrow y = \frac{1}{3}sinx - \frac{2}{3}x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}sinx - \frac{2}{3}x\right)$$

$$= \frac{1}{3}cosx - \frac{2}{3}$$ Ans.

2. $2x + 3y = siny$

Solution : দিয়া আছে,

$$2x + 3y = siny$$

$$\Rightarrow \frac{d}{dx}(2x + 3y) = \frac{d}{dx}siny$$

$$\Rightarrow 2 + 3\frac{dy}{dx} = cosy\frac{dy}{dx}$$

$$\Rightarrow (cosy - 3)\frac{dy}{dx} = 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{cosy-3}$$ Ans.

3. $ax + by^2 = cosy$

Solution : দিয়া আছে,

$$ax + by^2 = cosy$$

$$\Rightarrow \frac{d}{dx}(ax + by^2) = \frac{d}{dx}cosy$$

$$\Rightarrow a + 2by\frac{dy}{dx} = -siny\frac{dy}{dx}$$

$$\Rightarrow (2by + siny)\frac{dy}{dx} = -a$$

$$\Rightarrow \frac{dy}{dx} = -\frac{a}{2by+siny}$$ Ans.

4. $xy + y^2 = tanx + y$

Solution : দিয়া আছে,

$$xy + y^2 = tanx + y$$

$$\Rightarrow \frac{d}{dx}(xy + y^2) = \frac{d}{dx}(tanx + y)$$

$$\Rightarrow y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = sec^2x + \frac{dy}{dx}$$

$$\Rightarrow (x + 2y - 1)\frac{dy}{dx} = sec^2x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{sec^2x-y}{x+2y-1}$$ Ans.

5. $x^2 + xy + y^2 = 100$

Solution : দিয়া আছে,

$x^2 + xy + y^2 = 100$

$\Rightarrow \frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(100)$

$\Rightarrow 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\Rightarrow (x + 2y)\frac{dy}{dx} = -2x - y$

$\Rightarrow \frac{dy}{dx} = \frac{-2x-y}{x+2y}$ Ans.

6. $x^3 + x^2y + xy^2 + y^3 = 81$

Solution : দিয়া আছে,

$x^3 + x^2y + xy^2 + y^3 = 81$

$\Rightarrow \frac{d}{dx}(x^3 + x^2y + xy^2 + y^3) = \frac{d}{dx}(81)$

$\Rightarrow 3x^2 + 2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow (x^2 + 2xy + 3y^2)\frac{dy}{dx} = -3x^2 - 2xy - y^2$

$\Rightarrow \frac{dy}{dx} = -\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$ Ans.

7. $\sin^2 y + cosxy = \pi$

Solution : দিয়া আছে,

$\sin^2 y + cosxy = \pi$

$\Rightarrow \frac{d}{dx}(\sin^2 y + cosxy) = \frac{d}{dx}(\pi)$

$\Rightarrow 2sinycosy\frac{dy}{dx} - sinxy\frac{d}{dx}(xy) = 0$

$\Rightarrow 2sinycosy\frac{dy}{dx} - sinxy\left(y + x\frac{dy}{dx}\right) = 0$

$\Rightarrow (sin2y - xsinxy)\frac{dy}{dx} = ysinxy$

$\Rightarrow \frac{dy}{dx} = \frac{ysinxy}{sin2y-xsinxy}$ Ans.

8. $\sin^2 x + cos^2 y = 1$

Solution : দিয়া আছে,

$\sin^2 x + cos^2 y = 1$

$\Rightarrow \frac{d}{dx}(\sin^2 x + cos^2 y) = \frac{d}{dx}(1)$

$\Rightarrow \frac{d}{dx}(sin^2 x) + \frac{d}{dx}(cos^2 y) = 0$

$\Rightarrow 2sinxcosx + 2cosy.(-siny)\frac{dy}{dx} = 0$

$\Rightarrow sin2x - sin2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{sin2x}{sin2y}$ Ans.