Ordinary differential equations

A differential equation is an equation which contains derivatives.

Two types of differential equation- 1. Ordinary differential equations

2. partial differential equations

Here, we will focus only on ordinary differential equations.

Please revise what are--- order of differential equation and degree of differential equation.

Linear differential equation:: If the dependent variable and all of its derivatives occur in the first power without the product of the dependent variable and its derivatives(there cannot be any term which contains the product of the dependent variable and its derivative).

e.g- $\frac{dy}{dx} + xy = 0$ ……… Linear(1st order) y is the dependent variable and x is the independent variable.

$\frac{dy}{dx} - y = x$ ……………Linear(1st order)

$(\frac{dy}{dx})^2 + y = 0$ …………….Non-Linear

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x$ ………Linear(2nd order)

The general form of **first order linear** ordinary differential equation is $\frac{dy}{dx} + p(x)y = Q(x)$, p(x) & Q(x) are function of x or constants.

OR $y' + p(x)y = Q(x)$, $y' \equiv \frac{dy}{dx}$

[i]If Q(x) = 0, then, $\frac{dy}{dx} + p(x)y = 0$ is called homogeneous.

If Q(x) $\neq$ 0, then, $\frac{dy}{dx} + p(x)y = Q(x)$, it is called Non-homogeneous(inhomogeneous)

The general solution of first order linear differential equation is

$y \times I.F = \int (Q(x) \times I.F)dx + C$, (you do not need to memorise.)

Where $I.f = e^{\int p(x)dx}$ is called integrating factor.

** ***Solution of differential equation means relation between dependent variable and independent variable.***

This is homogeneous.

Q.1. $\frac{dy}{dx} = ky$, k is a Constant

Sol $\frac{dy}{dx} = ky$

$\Rightarrow \frac{dy}{dx} - ky = 0$ ——— (1)

$\therefore P(x) = -k$

$I.F = e^{\int P(x)dx} = e^{\int -k dx} = e^{-kx}$

multiply eqn. (1) by I.F,

$\left(\frac{dy}{dx} - ky\right) I.F = 0$

$\Rightarrow \left(\frac{dy}{dx} - ky\right) e^{-kx} = 0$

$\Rightarrow \frac{d}{dx}\left(y e^{-kx}\right) = 0$

$\Rightarrow y e^{-kx} = C$, $C \to$ Constant

$\Rightarrow \boxed{y = Ce^{kx}}$

This is the Solution

(you can also solve by method of separation of variables)

This is non-homogeneous

Q2. $L\frac{di}{dt} + iR = E$, where L, R, E are constant with initial condition $i=0$ at $t=0$.

Sol $L\frac{di}{dt} + iR = E$

$\Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ ——— (i)

Here, $P(x) = \frac{R}{L}$ & $Q(x) = \frac{E}{L}$

$\therefore$ I.F $= e^{\int P(x)dx} = e^{\int \frac{R}{L}dt} = e^{\frac{Rt}{L}}$

Now, multiplying both sides of eqn. (i) by I.F,

$\left(\frac{di}{dt} + \frac{R}{L}i\right)\text{I.F} = \frac{E}{L} \times \text{I.F}$

$\Rightarrow \frac{d}{dt}(i\,\text{I.F}) = \frac{E}{L} \times \text{I.F}$

$\Rightarrow i \times \text{I.F} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt + c_1$, $c_1 \rightarrow$ Constant

$\Rightarrow i \times e^{\frac{Rt}{L}} = \frac{E}{L} \times \frac{L}{R} e^{Rt/L} + c_1$

$\Rightarrow i = \frac{E}{R} + c_1 e^{-\frac{Rt}{L}}$ ——— (ii)

Initial Condition,

$i=0$ at $t=0$

$\therefore 0 = \frac{E}{R} + c_1$

$\Rightarrow c_1 = -\frac{E}{R}$

From (ii)

$i = \frac{E}{R} - \frac{E}{R} e^{-Rt/L}$

$\Rightarrow \boxed{i(t) = \frac{E}{R}\left(1 - e^{-Rt/L}\right)}$ This is the required Solution.

Find the general solution.

1. $\frac{dy}{dx} = ax + by$
2. $xy' + 2y = 12x$
3. $y - y' - xy^2 = 0$
4. $y' + 3y - x - e^{-2x} = 0$

References:

1. Introduction to mathematical physics, Charlie Harper, PHI
2. Principles of mathematical physics, S P Kuila, NCBA