টোকা-২

 # অনুকলনৰ কিছু ধৰ্ম:

(১) $f(x)$ *আৰু* $g(x)$ দুটা x-ৰ ফলন হ'লে

$$\int \{f(x) + g(x)\}dx = \int f(x)dx + \int g(x)dx$$

উদাহৰণ: $\int (\sin x + \cos x)dx = \int \sin x\, dx + \int \cos x\, dx$

(২) $f(x)$ *এটা* ফলন আৰু k এটা ধ্ৰুবক হ'লে

$$\int kf(x)dx = k\int f(x)dx$$

উদাহৰণ: $\int 5\sin x\, dx = 5\int \sin x\, d$

(৩) $f(x)$ *আৰু* $g(x)$ দুটা x-ৰ ফলন আৰু c_1, c_2 দুটা ধ্ৰুবক হ'লে

$$\int \{c_1 f(x) + c_2 g(x)\}dx = c_1\int f(x)dx + c_2\int g(x)dx$$

উদাহৰণ: $\int (4\sin x + 5\cos x)dx = 4\int \sin x\, dx + 5\int \cos x\, dx$

Ex. নিৰ্ণয় কৰা।

(i) $\int (4e^{4x}dx + 5)dx$ **(ii)** $\int x^2\left(1 - \frac{1}{x^2}\right)dx$ **(iii)** $\int \frac{x^3+5x^2-4}{x^2}dx$

(iv) $\int \sec x\,(\sec x + \tan x)dx$ **(v)** $\int \frac{\sec^2 x}{cosec^2 x}dx$ **(vi)** $\int \frac{2-3\sin x}{\cos^2 x}dx$

(vii) $\int \frac{x^3-x^2+x-1}{x-1}dx$

Solution:

(i) $\int (4e^{4x}dx + 5)dx$

$= 4\int e^{4x}dx + 5\int dx$

$= 4\frac{e^{4x}}{4} + 5x + C$

$= e^{4x} + 5x + C$

(ii) $\int x^2\left(1 - \frac{1}{x^2}\right)dx$

$$= \int x^2 dx - \int 1\, dx$$
$$= \int x^2 dx - \int dx$$
$$= \frac{x^3}{3} - x + C$$

(iii)

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$
$$= \int (x + 5 - \frac{4}{x^2}) dx$$
$$= \frac{x^2}{2} + 5x - \left(-\frac{4}{x}\right) + C$$
$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

(iv) $\int \sec x\, (\sec x + \tan x) dx$

$$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$$
$$= \tan x + \sec x + C$$

(v) $\int \frac{\sec^2 x}{cosec^2 x} dx$

$$= \int \frac{1/\cos^2 x}{1/\sin^2 x} dx$$
$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$
$$= \int \tan^2 x\, dx$$
$$= \int (\sec^2 x - 1) dx$$
$$= \int \sec^2 x\, dx - \int dx$$
$$= \tan x - x + C$$

(vi) $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

$$= \int \left(\frac{2}{\cos^2 x} - 3\frac{1}{\cos x}\frac{\sin x}{\cos x}\right) dx$$

$$= \int (2\sec^2 x - 3\sec x \tan x)dx$$
$$= 2\int \sec^2 x\, dx - 3\int \sec x \tan x\ dx$$
$$= 2\tan x - 3\sec x + C$$

(vii) $\int \frac{x^3-x^2+x-1}{x-1}dx$

$$= \int \frac{(x-1)(x^2+1)}{x-1}dx$$
$$= \int (x^2+1)dx$$
$$= \frac{x^3}{3} + x + C$$

$x^3 - x^2 + x - 1$
$= x^2(x-1) + (x-1)$
$= (x-1)(x^2+1)$

Ex: যদি $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ যাতে $f(2) = 0$ তেন্তে $f(x)$ নিৰ্ণয় কৰা ।

Solution: আমাক দিয়া আছে,

$$\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$$

এতেকে, $f(x) = \int \left(4x^3 - \frac{3}{x^4}\right)dx$

$$= 4\int x^3 dx - 3\int x^{-4}dx$$
$$= 4\frac{x^4}{4} - 3\frac{x^{-4+1}}{-4+1} + C$$
$$= x^4 - 3\frac{x^{-3}}{-3} + C$$

$\Rightarrow f(x) = x^4 + \frac{1}{x^3} + C \longrightarrow (1)$

দিয়া আছে, $f(2) = 0$ । এতেকে (1)–ৰ পৰা –

$$0 = 2^4 + \frac{1}{2^3} + C$$
$$\Rightarrow C = -16 - \frac{1}{8} = -\frac{129}{8}$$

এতিয়া (1) নং সমীকৰণত $C = -\frac{129}{8}$ বহুৱাই আমি পাওঁ–

$$f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$