$$\ln W = \sum_{i=1}^{k} \left[g_i \ln g_i - \cancel{g_i} - n_i \ln n_i + \cancel{n_i} - (g_i - n_i)\ln(g_i - n_i) + \cancel{g_i} - \cancel{n_i} \right]$$

$$\ln W = \sum_{i=1}^{k} \left[g_i \ln g_i - n_i \ln n_i - (g_i - n_i)\ln(g_i - n_i) \right]$$

Differentiating both sides,

$$\delta(\ln W) = \sum_{i=1}^{k} \left[-\cancel{n_i}\frac{1}{\cancel{n_i}}\delta n_i - \ln n_i\,\delta n_i + \cancel{(g_i - n_i)}\frac{1}{\cancel{(g_i - n_i)}}\delta n_i \right.$$
$$\left. + \ln(g_i - n_i)\,\delta n_i \right]$$

, Here, g_i is not subjected to variation i.e. its differentiation is zero.

$$\delta(\ln W) = \sum_{i=1}^{k} \left[\ln(g_i - n_i) - \ln n_i \right] \delta n_i$$

$$= \sum_{i=1}^{k} \left[\ln \frac{(g_i - n_i)}{n_i} \right] \delta n_i$$

To get the state of maximum thermodynamic probability,

$$\delta(\ln W) = 0$$

$$\sum_{i=1}^{k} \left[\ln \frac{g_i - n_i}{n_i} \right] \delta n_i = 0$$

$$-\sum_{i=1}^{k} \left[\ln \frac{n_i}{g_i - n_i} \right] \delta n_i = 0$$

$$\sum_{i=1}^{k} \left[\ln \frac{n_i}{g_i - n_i} \right] \delta n_i = 0 \qquad \text{——— } (3)$$

We have, the conditions of Constraint,

$\Rightarrow$ $n = n_1 + n_2 + \cdots + n_k =$ Constant

$$\therefore \delta n = \sum_{i=1}^{k} \delta n_i = 0 \qquad \text{——— } (4)$$

$\Rightarrow$ $E = n_1 E_1 + n_2 E_2 + \cdots + n_k E_k =$ Constant

$$\therefore \delta E = \sum_{i=1}^{k} E_i\, \delta n_i = 0. \qquad \text{——— } (5)$$